**Math Placement Test Recommendations**

Dr. Susan Lauer, Dr. Jason Joyner, and Dr. Greg Bell

Math SAT scores were primarily used to place students into math classes at Wingate University in the past, matching the best class for each student with available seats in classes on the schedule. WU typically used the criteria of 500 on the SAT as an indicator that students were ready for any class appropriate to their major. Other factors were considered if the SAT score was below 500. General minimum standards such as SAT or ACT scores and high school GPA seem to be reliable for placement into Math 115, Precalculus, but none of these scores indicate which students have taken a precalculus course, the prerequisite for placement into Calculus I or Business Calculus.

In recent years, Dr. Greg Bell was spending part of his summer physically combing through transcripts from every new incoming student to determine which students had previously succeeded in a course in precalculus. This was already an incredibly long process with just 500 students, so with the recent steady increase in enrollment, the Math Department has sought to improve student math placement by instituting a math placement test for all incoming students.

There is no end of course test for precalculus in high schools, so content and rigor varies widely. Scores of 500 or better on the Math SAT and 20 or better on the ACT are very good indicators for success in most math classes, but we currently admit many students who do not achieve these minimum scores. The placement test can distinguish between those students who truly need a more introductory college math course and those who are ready for college level mathematics.

There have been a few challenges with this placement process, as our transcript system doesn’t currently keep track of recommended placement. This leads to confusion for both advisors and advisees. For instance, students who need calculus their second semester need to complete a prerequisite override form that requires a manual lookup of the placement score. This isn’t difficult, but it is another step for registration that could be eliminated.

The placement test was included in the application process for incoming students for Fall 2017 and Fall 2018. We enlisted Dr. Dwight Lauer of Silvics Analytics in Wingate to analyze the data for Fall 2017. Dwight has been a data analyst in various capacities for more than 35 years. He ran probability models on the various math classes comparing SAT, ACT, High School GPA, and placement test results. The appendix contains this detailed analysis.

The most surprising result of this analysis is that success in Inferential Statistics, Math 209, is strongly linked to placement test results. A student is much more likely to succeed in this course if they have a placement test score of more than 19 or if they’ve taken a math course on campus before attempting Math 209.

The historical standardized test criteria was found to be adequate for placement into Math 115. Results for Math 117 and 120 placement were very similar. Higher scores on the placement test significantly targetted the probability of success in these courses.

The columns of the table below are the recommended criteria based on this analysis.

**Prerequisite Course Recommendations**

|  |  |  |  |
| --- | --- | --- | --- |
| **Course** | **Criteria Group 1** | **Criteria Group 2** | **Criteria Group 3** |
| **Math 112** |  | Math SAT < 500  **AND** Placement < 20 | ACT < 20  **AND** Placement < 23 |
| **Math 115** | Math SAT ≥ 500  **OR** ACT ≥ 20  **OR** Math 112 | Math SAT < 500  **AND** 20 ≤ Placement < 23 | ACT < 20  **AND** 23 ≤ Placement < 26 |
| **Math 117**  **Math 120** | Placement ≥ 26  **OR** Math 115 | Math SAT < 500  **AND** Placement ≥ 23 | ACT < 20  **AND** Placement ≥ 26 |
| **Math 209** | Placement ≥ 20  **OR** Math 112  **OR**  Math 115  **OR**  Math 116  **OR**  Math 117  **OR** Math 120 |  |  |

The prerequisite for Business Calculus, Math 117, is currently the same as Calculus I, Math 120. The math department should reconsider a 3-hour precalculus course catering to students who will not need Math 120 for their major.

The math department recommends that the placement test scores be included as part of each student’s Unofficial Transcript. We further recommend that the Academic Catalog be updated to include the table above as Prerequisite Course Requirements.

**Appendix**

**Analysis of Placement Test Scores**

**Dr. Dwight Lauer**

**Sivics Analytics**

**Wiungate, NC**

**Summary points**

Models that predicted success for Math 112, 115, 120 as a function of pre-test score, HSGPA, or ACT/MSAT scores produced similar results. The HSGPA produced the best fit statistics but HSGPA is not always available when placements are made.

Pre-test score was a significant predictor of success for 117, 120, and 209 after accounting for ACT/MSAT scores. This indicates that pre-test scores improved prediction of success even when ACT/MSAT scores were available.

Math 115 (Pre-calculus) success probabilities were higher (0.82 vs. 0.57) for students that took Math 115 when 120 was recommended based on pre-test scores.

Math 115 best predictors of success were MSAT or ACT scores, not pre-test scores. This was likely due to censoring (the placement of students with higher scores in higher level courses). Figures S-1 and S-2 show the probability of success as it relates to MSAT or ACT scores.

Math 117 (Business calculus) success probability was higher (0.79 vs. 0.45) for students with the pre-test score recommendation to take Math 120. Both standardized test scores and pre-test scores were significant predictors of success. The use of a pre-test score threshold with consideration of discrepancies between this score and MSAT/ACT score (Tables S-1 and S-2) might improve placement of students

Math 120 (Calculus) success probability was higher (0.81 vs. 0.45) for students with the pre-test score recommendation to take Math 120. Both standardized test scores and pre-test scores were significant predictors of success. The use of a pre-test score threshold with consideration of discrepancies between this score and MSAT/ACT score (Tables S-3 and S-4) might improve the placement of students

Math 209 (Statistics) success probability was related to pre-test scores only. ACT/MSAT scores were not significant when pre-test score was included in the analysis. Figure S-3 relates the probability of success to pre-test score.

Math 209 success was significantly higher when students had previously taken Math 120 (0.88 vs. 0.67) but not 115 or 117. These results are probably not definitive due to the small sample size and correlated factors.



Figure S-1. Average predicted Math 115 success rates with respect to MSAT score. The number in the circle is the sample size for each score.



Figure S-2. Figure S-1. Average predicted Math 115 success rates with respect to MSAT score. The number in the circle is the sample size for each score.



Figure S-3. Average predicted Math 209 success rates with respect to pre-test score. The number in the circle is the sample size for each score.

Table S-1. Math 117 predicted success probabilities with respect to pre-test and MSAT scores.





Table S-2. Math 117 predicted success probabilities with respect to pre-test and ACT scores.





Table S-3. Math 120 predicted success probabilities with respect to pre-test and MSAT scores.





Table S-4. Math 120 predicted success probabilities with respect to pre-test and ACT scores.





**Methods**

The objectives were to 1) examine success rates (probability of success) for Math 112, 115 and 120 with respect to recommended placement and 2) relate success rate to placement test scores, high school grade point average (HSGPA), and SAT/ACT scores to determine if placement tests are required to improve placement.

Only the first-time grade for each student was selected for Math 112, 115 and 120 to avoid cases where the course was not the first in the series taken or the student repeated a course. A grade of A, B, or C was considered a “Success”. The three categories of recommended course placement where 1) taking the recommended course (Yes), 2) not taking the recommended course and taking a higher-level course (No), and 3) not taking the recommended course but taking a lower-level course (Lower).

A generalized linear model (SAS PROC GLIMMIX) was used to relate the logit of success probability to course taken, course recommendation category, placement test scores, HSGPA, and ACT/MSAT scores. Clustering within class sections was addressed by treating class sections as a random effect. Fixed effects included classification variables of course taken and recommendation category. Covariates tested with categorical effects were total pre-test score, the ACT score or math SAT score, and high school grade point average (HSGPA). Orthogonality of standardized test scores was maintained by using only the math SAT score when a student had both ACT and SAT scores. Models were compared in terms of log-likelihood, conditional likelihood, and ROC AUC scores. The ROC AUC (area under the curve) of 1 would be perfect agreement in a frequency table, 0.5 would indicate an uninformative classifier. Values close to 0.8 or higher are usually considered acceptable rates but this is a subjective decision.

The response variable of interest is the number of successes (students passing) expressed as the probability of success, where The distribution of is the probability of success for each category. A generalized linear model for this investigation is:

(1)

where µ is the intercept, αi is the random effect of the class (section of a math course), cj is the fixed effect of math course taken, rk is the fixed effect of the recommendation classification (Yes, No, Lower), crjk is the course and recommendation interaction, and (αcr)ijk is the random effect interaction of class section, course, and recommendation. The vector of random class section effects is distributed  and the vector of random class, course, and recommendation interaction is distributed . This model is modified by replacing the course recommendation effect with covariates of pre-test scores, ACT/MSAT scores, and HSGPA. Further, the one observation in the “Lower” recommendation class for math course 112 was excluded for the categorical model because a sample size of 1 in that class is uninformative.

Hypothesis tests were partitioned using model (1) to account for missing categories. The dataset contains observations for Math 115 and Math 120 with Yes and No recommendation categories. These cells form a crossed classification that allow the following means comparisons; 1) Comparison of 115 and 120 success rates averaged over Yes & No categories, 2) Comparison of Yes and No recommendation success rates averaged over 115 and 120, and 3) the interaction of these means. The fourth comparison possible was that between course 115 “Lower versus “Yes” success rates.

Finally, model (1) was fit separately by course (c-term dropped), with categories or categories replaced with test scores, to obtain the best course-based estimates. This was expanded to include Math 117 and 209. A categorical analysis using the last course taken (115, 117, 120, or None) was performed for Math 209.

**Categorical Analysis Results**

Students had the options of taking the recommended course (Yes), taking a course more advanced than the recommendation (No), or taking a lower level course than recommended (Lower). The sample size with respect to recommendation and course taken are listed in Table 1 and the raw success rates by recommendation and course taken are summarized in Table 2. The raw scores do not account for the clustering by class.

Predicted probabilities of success using the fitted categorical model (1) with approximate standard errors are listed in Table 3. In terms of hypothesis tests, average success rates for 115 and 120 of 0.66 and 0.65, respectively, were not significantly different (p=0.967), average success rates for Yes and No (for 115 & 120) of 0.71 and 0.60, respectively, were not significantly different (p= 0.443), and there was a significant interaction of the recommendation and course effect (p=0.038). This interaction was the result of a higher success rate for the “No” than “Yes” category for 115 and a higher success rate for the “Yes” than “No” category for 120 (Table 3). The conclusion is that the recommendation is of value for Math 120 but that students that choose to take 115 against recommendation are motivated or have higher ability than reflected in the pre-test scores. Students taking 115 when 120 was recommended had a significantly higher success rate (p= 0.018) of 0.82 compared to 0.57 for students for which 115 was recommended.

**Relating Test Scores to Success**

**Total pre-test score**: Probability of getting a C grade or better was related to pre-test score by replacing the recommendation category in Model (1) with actual pre-test score. Initial models with pre-test score found that separating the pre-calc score from the total score was not significant (p=0.527) and there were no significant interactions between scores and course taken (p=0.216). The course effect was not significant (p=0.606) but was included in the model because it is a design variable. Model fit statistics are listed in Table 4. The relationship between success averages, course taken, and pre-test scores are shown in Figure 1. Model parameter estimates are listed in Table 5.

**HSGPA:** The model was fit with HSGPA in place of pre-test score and with both HSGPA and pre-test score. These HSGPA models were the best models in terms of log-likelihood and ROC fit statistics (Table 4). The pre-test score component was significant (p=0.002) and indicated that success increased with pre-test score even when HSGPA was included in the model. The relationship between success averages, course taken, and HSGPA are shown in Figure 2.

**ACT/MSAT**: The model was fit with ACT and math SAT scores in place of pre-test score and with pre-test score included. Fit statistics for these models were rather close to those for pre-test score alone (Table 4). The pre-test score component was significant (p=0.003) when included in the ACT/MSAT model and indicated success increased with pre-test score. The relationship between success averages, course taken, and MSAT or ACT scores are shown in Figures 3 and 4, respectively.

**Course-Level Analysis**

The analysis was performed on a course by course basis to obtain the best estimates for each course. Math 117 was included to determine if test scores were predictors of success. Math 209 was examined with respect to last math course taken and to relate test scores to success when this was the first course taken.

**Math 115 (Pre-calculus):** Success rates were significantly higher only for students taking 115 when a higher math course was recommended. After accounting for ACT/MSAT scores, the pre-test score was not a significant predictor (Table 6). This may be due to the retrospective design since students starting in this course are usually there because they score in the lower range of the pre-test. A 70% or higher chance of success is achieved by students that score 450 or higher on the MSAT and greater than 20 on the ACT.

**Math 117 (Business calculus):** Students recommended to take 120 (or 117) using pre-test scores had a higher success probability than those with a lower course level recommendation (0.79 vs. 0.45). Pre-test score was significant after accounting for ACT or MSAT scores. The use of a pre-test score threshold with consideration of discrepancies between this score and MSAT/ACT score may help placement of students.

**Math 120 (Calculus):** Pre-test placement of students was effective with an average success rate of 0.81 for students recommended to this course (versus 0.45). However, both standardized test scores and pre-test scores were significant predictors (Table 7). The use of a pre-test score threshold with consideration of discrepancies between this score and MSAT/ACT score may help placement of students.

**Math 209 (Statistics)**: Analysis was performed in two ways. The first was a categorical analysis that accounts for the last course taken. The second was an analysis of success when this was taken as the first math course. Last course taken categories were Math 115, Math 117, Math 120, and None. Success rate for taking a math course before 209, after accounting for ACT/MSAT and pre-test scores, was significant only for those taking math 120 before 209 (0.88 for Math 120 vs. 0.67 for None, Table 9). There were only 20 (out of 401) students that had taken 115 or 117 before 209. These results are not very conclusive due to the small sample size. The analysis of 209 taken as the first course found that pre-test score was significant but ACT/MSAT scores were not. A final model was constructed without ACT/MSAT (Table 9).

Table 1. Distribution of observations with respect to course taken and course recommended.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Course taken | | |  |
| Recommendation | 112 | 115 | 120 | Total |
| 112 | 21 | 8 | 1 | 30 |
| 115 | 0 | 38 | 11 | 49 |
| 120 | 1 | 57 | 155 | 213 |
| Total | 22 | 103 | 167 | 292 |

Table 2. Sample size (n) and success rates (%) by course recommendation and course taken. The “Lower” classification indicates that a course lower than that recommended was taken.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Recommendation | 112 | |  | 115 | |  | 120 | |
| Classification | (n) | (%) |  | (n) | (%) |  | (n) | (%) |
| No | 0 | - |  | 8 | 75 |  | 12 | 50 |
| Yes | 21 | 62 |  | 38 | 55 |  | 155 | 77 |
| Lower | 1 | 100 |  | 57 | 79 |  | 0 | - |

Table 3. Predicted probability of success with approximate standard error in parentheses. The course 112, “Lower” recommendation was excluded from the analysis because there was only 1 student in that classification.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Recommended | 112 | 115 | 120 | Avg. of 115 & 120 |
| No | - | 0.74 (0.18) | 0.45 (0.17) | 0.60 (0.14) |
| Yes | 0.62 (0.21) | 0.57 (0.12) | 0.81 (0.05) | 0.71 (0.06) |
| Lower | - | 0.82 (0.07) | - |  |
| Avg. of Yes & No |  | 0.66 (0.13) | .65 (0.09) |  |

Table 4. Significance of model effects and model fit statistics.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | df |  |  | HSGPA | | ACT/MSAT | |  |
| Effect |  |  | Score |  | w/score |  | w/score |  |
|  |  |  | ------------ Prob. of a greater F-value ------------ | | | | |  |
| Pre-test score (algebra + pre-calc) | 1 |  | <0.001 | - | 0.002 | - | 0.003 |  |
| MSAT | 1 |  | - | - | - | <0.001 | <0.001 |  |
| ACT | 1 |  | - | - | - | <0.001 | <0.001 |  |
| HSGPA | 1 |  | - | <0.001 | <0.001 | - | - |  |
| Course differences | 2 |  | 0.606 | 0.640 | 0.198 | 0.984 | 0.516 |  |
| Fit statistics |  |  |  |  |  |  |  |  |
| -2 log L |  |  | 312.2 | 280.3 | 259.5 | 306.4 | 296.6 |  |
| -2 log L(Success | r. effects) |  |  | 285.1 | 244.9 | 232.3 | 283.4 | 270.8 |  |
| ROC AUC |  |  | 0.78 | 0.84 | 0.86 | 0.77 | 0.79 |  |

Table 5. Parameter estimates for each model. Probability of success can be calculated as 1/(1+exp(-***B***x)). For example, using the score model for Math 120 with a pre-test score of 26 yields ***B***x= -1.9519 + 0.0 + 0.1281 x 26 = 1.3787 and probability of success = 1/(1+exp(-1.3787)) = 0.799.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | Score | HSGPA | HSGPA + Score | ACT/MSAT | ACT/MSAT + Score |
| Intercept |  | -1.9519 | -7.2999 | -9.1593 | -4.1874 | -5.4753 |
| Course | 112 | 0.9633 | 0.5813 | 1.7963 | -0.01056 | 1.0223 |
|  | 115 | 0.3886 | 0.4996 | 0.9945 | 0.08389 | 0.5067 |
|  | 120 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Pre-test score |  | 0.1281 |  | 0.1025 |  | 0.09251 |
| HSGPA |  |  | 2.2265 | 2.0505 |  |  |
| ACT/MSAT | ACT |  |  |  | 0.2499 | 0.2015 |
|  | MSAT |  |  |  | 0.01048 | 0.00857 |

Table 6. Summary of 115 analysis.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | Categorical |  | ACT/MSAT/PS |  | ACT/MSAT |
| Effect |  | Prob. > F-value | | | | |
| Yes vs. No |  | 0.468 |  | - |  | - |
| Low vs. Yes |  | 0.019 |  | - |  | - |
| ACT |  | - |  | 0.004 |  | 0.001 |
| MSAT |  | - |  | 0.002 |  | <0.001 |
| Pre-score |  | - |  | 0.243 |  | - |
| Fit statistics |  |  |  |  |  |  |
| -2 LL|r. effects |  | 101.90 |  | 87.14 |  | 88.76 |
| AUC stat |  | 0.79 |  | 0.85 |  | 0.85 |
| Means and parameter estimates | | | | | | |
|  | N | Success |  | Estimate |  | Estimate |
| Category |  | % (se) |  | - |  | - |
| Lower | 57 | 82 ( 8) |  | - |  | - |
| No | 8 | 74 (19) |  | - |  | - |
| Yes | 38 | 58 (13) |  | - |  | - |
| Covariates |  |  |  |  |  |  |
| Intercept |  |  |  | -6.6030 |  | -6.0963 |
| ACT | 46 |  |  | 0.3112 |  | 0.3407 |
| MSAT | 57 |  |  | 0.01422 |  | 0.01548 |
| Pre-score | 103 |  |  | 0.05552 |  | - |

Table 7. Summary of 120 analysis.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | Categorical |  | ACT/MSAT/PS |  |  |
| Effect |  | Prob. > F-value | | | | |
| Yes vs. No |  | 0.019 |  | - |  |  |
| ACT |  | - |  | 0.042 |  |  |
| MSAT |  | - |  | 0.035 |  |  |
| Pre-score |  | - |  | 0.014 |  |  |
| Fit statistics |  |  |  |  |  |  |
| -2 LL|r. effects |  | 164.64 |  | 156.65 |  |  |
| AUC stat |  | 0.76 |  | 0.76 |  |  |
| Means and parameter estimates | | | | | | |
|  | N | Success |  | Estimate |  |  |
| Category |  | % (se) |  | - |  |  |
| No | 12 | 45 (16) |  | - |  |  |
| Yes | 155 | 81 ( 5) |  | - |  |  |
| Covariates |  |  |  |  |  |  |
| Intercept |  |  |  | -4.805 |  |  |
| ACT | 61 |  |  | 0.154 |  |  |
| MSAT | 106 |  |  | 0.006162 |  |  |
| Pre-score | 167 |  |  | 0.1085 |  |  |

Table 8. Summary of 117 analysis.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | Categorical |  | ACT/MSAT/PS |  |  |
| Effect |  | Prob. > F-value | | | | |
| Yes vs. No |  | 0.006 |  | - |  |  |
| ACT |  | - |  | 0.004 |  |  |
| MSAT |  | - |  | 0.001 |  |  |
| Pre-score |  | - |  | 0.008 |  |  |
| Fit statistics |  |  |  |  |  |  |
| -2 LL|r. effects |  | 95.53 |  | 54.99 |  |  |
| AUC stat |  | 0.75 |  | 0.92 |  |  |
| Means and parameter estimates | | | | | | |
|  | N | Success |  | Estimate |  |  |
| Category |  | % (se) |  | - |  |  |
| No | 22 | 45 (11) |  | - |  |  |
| Yes | 66 | 79 ( 6) |  | - |  |  |
| Covariates |  |  |  |  |  |  |
| Intercept |  |  |  | -13.2481 |  |  |
| ACT | 41 |  |  | 0.5108 |  |  |
| MSAT | 47 |  |  | 0.02196 |  |  |
| Pre-score | 88 |  |  | 0.1671 |  |  |

Table 9. Summary of 209 analysis.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | Last Course Analysis |  | First Course Analysis | | |
|  |  | ACT/MSAT/PS |  | ACT/MSAT/PS |  | PS |
| Last Course Taken |  | Prob. > F-value | | | | |
| Math 115 vs. None |  | 0.911 |  | - |  | - |
| Math 117 vs. None |  | 0.364 |  | - |  | - |
| Math 120 vs. None |  | 0.015 |  | - |  |  |
|  |  |  |  |  |  |  |
| Test Scores |  |  |  |  |  |  |
| ACT |  | <0.001 |  | 0.869 |  | - |
| MSAT |  | <0.001 |  | 0.777 |  | - |
| Pre-score |  | <0.001 |  | 0.007 |  | 0.005 |
| Fit statistics |  |  |  |  |  |  |
| -2 LL|r. effects |  | 439.27 |  | 100.46 |  | 101.35 |
| AUC stat |  | 0.76 |  | 0.81 |  | 0.81 |
| Means and parameter estimates | | | | | | |
|  | N | Success | N | Estimate |  | Estimate |
| Category |  | % (se) |  |  |  |  |
| Math 115 | 14 | 66 (14) |  |  |  |  |
| Math 117 | 6 | 46 (23) |  |  |  |  |
| Math 120 | 36 | 88 ( 5) |  |  |  |  |
| None | 345 | 67 ( 3) |  |  |  |  |
| Covariates |  |  |  |  |  |  |
| Intercept |  | -4.9867 |  | -0.6293 |  | -0.9372 |
| ACT | 159 | 0.1994 | 137 | -0.01581 |  | - |
| MSAT | 242 | 0.007495 | 210 | -0.00105 |  | - |
| Pre-score | 401 | 0.07726 | 347 | 0.1071 |  | 0.1006 |



Figure 1. Probability of getting a C grade or higher with respect to pre-test total score and math course taken. Number in circle is the sample size.



Figure 2. Probability of getting a C grade or higher with respect to High School GPA and course taken. Number in circle is sample size.



Figure 3. Probability of getting a C grade or higher with respect to Math SAT score and course taken. Number in circle is sample size.



Figure 4. Probability of getting a C grade or higher with respect to ACT score and course taken. Number in circle is sample size.